Near-Polar Satellite Constellations for Continuous Global Coverage

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New methods are described for designing circular near-polar orbit satellite constellations that can provide continuous single- and double-global Earth coverage. These methods are based on a generalization of the known polar-phased satellite constellations and are given as a general class of constellations that includes the polar-phased satellite constellations as a bound for this class. The meshing geometry and constraint relations are described. Sets of optimal constellations are presented. The new constellations are very comparable in terms of numbers of satellites vs orbit altitude to the best polar constellations. In fact, the two-plane near-polar constellations require the same orbit altitudes as the best polar constellations having the same total numbers of satellites. For single-global coverage, the phased-polar constellations are slightly better than the near-polar constellations. In most cases, the opposite situation occurs for double coverage. The results are compared with the best Walker and unphased (polar and inclined) constellations. A significant advantage of near-polar constellations is the possibility of being able to select an orbit inclination in a wide range with only slight changes in orbit altitude.

Nomenclature

 C_i = half-street width of continuous j-fold coverage; Fig. 2

h = orbit altitude; Fig. 1

= orbit inclination, $\pi/2 \ge i \ge i_{\min}$

 $i_{\min} = \min_{i=1}^{\infty} i_{\min}$

j = level of coverage provided by satellites in the same orbit plane

 k = level of multiple coverage provided by overlapping streets-of-coverage

 $N = \text{total number of satellites}, P \times S$

n = level of multiple coverage, $j \times k$

P = number of orbit planes

 R_e = spherical radius of the Earth, 6378 km

S = number of satellites in each plane

 α = elevation angle; Fig. 1

 Δλ = angular separation between the ascending nodes for adjacent corotating orbit planes

 $\Delta \lambda^*$ = angular separation between the ascending nodes of the first and Pth orbit planes

 $\Delta \varphi$ = in-plane satellite phasing angle, $2\pi/S$

 $\Delta \varphi^*$ = interplane satellite phasing angle

 θ = coverage angle; Fig. 1

Introduction

THE problems of identifying the minimum number of satellites in circular orbit constellations that provide continuous global coverage have become of increasing importance in recent years due to global communication applications. Continuous single- and multiple-global coverage of the Earth's surface by satellite constellations in circular orbits has been examined by many authors since about 1960. Contributions were made at an early stage in Refs. 2–4, among others. To our knowledge, the first introduction of kinematically regular constellations is given by Mozhaev. Similar constellations have been developed independently by Walker. The introduction of kinematically regular constellations. These constellations are better known as Walker-typeor Walker constellations. In the past 20 years, some new methods have been developed for the design and analysis of satellite constellations. Methods using street-of-coverage

techniques with arbitrary and optimal interplane satellite phasing have been presented by Rider^{12,13} and Adams and Rider.¹⁴ Constellations for continuous global, single above-the-horizon coverage and global below-the-horizon coverage have been considered by Hanson and Linden.¹⁶ Gutenev¹⁸ has studied communication satellite constellations with intersatellite links. The expanded tables of Walker constellations for continuous single and multiple coverage for 5–100 satellites and all numbers of orbit planes have been given by Lang^{15,22} and Lang and Adams.²³ Most constellationstudies have concentrated on circular orbits with a common inclination.^{2–19,21–23} Yuan and Matsushima²⁰ have developed the hybrid constellations in which polar orbits and inclined orbits are both used.

As a rule, the satellite constellations have been compared in terms of numbers of satellites vs orbit altitude.^{2–20} It is known^{13,15,22,23} that the polar-phased satellite constellations are more efficient than Walker constellations^{6,11} when the total number of satellites is greater than approximately 20 for continuous single-global coverage. For double-global coverage, the best results have been obtained by Lang^{15,22} and Lang and Adams²³ for the Walker constellations. It should be noted that there are other complex criteria related to launch vehicle capability, crosslinking, sparing strategy.²³

Polar satellite constellations are not the most suitable constellations for some practical applications. As an example, the phased-polar satellite constellations required an approach to the satellite collision avoidance near the poles. ¹⁴ The selection of the near-polar inclination, i.e., 86.4 deg, for the IRIDIUM® constellation is based on the need to maximize the stability of the orbits and to maximize the closest approach distance as satellites near the poles. ²⁴ Launch azimuths can have limitations placed on them to keep launch vehicles from overflying populated areas; as a consequence, the associated orbital inclinations can be restricted as well. ²¹ Some constellations can use predefined nonpolar orbit types (for example, meridian orbits¹⁹ or well-known sun-synchronous orbits).

The present analysis identifies new families of circular orbit satellite constellations in which there is a wide range of inclination that can be selected. The term *near-polar satellite constellations*, as used here, implies a general class of constellationsthat includes the polar-phased satellite constellations¹⁴ as a bound for this class. In a sense, we extend the preceding studies^{12,14} by effecting a continuous transformation from the original polar constellations to a set of new constellations. These new constellations and the polar-phased constellations are similar in terms of the total number of satellites required for continuous coverage vs orbit altitude. The two-plane near-polar constellations require the same orbit altitude for a wide range of orbit inclinations, as is required for the polar-phased constellations. For single-global coverage, the phased-polar constellations are slightly

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better than the new constellations. However, in most cases, for double continuous global coverage, the near-polar constellations are slightly more efficient than the original polar constellations. In a sense, the new constellationspresented herein extend the knowledge base of constellations that can provide continuous global coverage to near-polar orbits.

Polar-Phased Satellite Constellations

In this section we begin by considering known facts related to coverage geometry and polar-phased satellite constellations. ^{12,14} We will make some standard assumptions: 1) the Earth is considered a round body; 2) all satellites in a constellation will be at the same altitude, with the same number of satellites in each orbit plane; and 3) all orbit planes in a constellation will have the same orbit inclination.

Figure 1 shows a typical satellite coverage region on the Earth. The satellite is located at orbital height h, and the projection of the footprint onto the Earth's surface defines a coverage angle θ . The well-known relation between the coverage angle θ , the orbit altitude h, and the elevation angle α is given by

$$\theta = \cos^{-1}\{[R_e/(R_e + h)]\cos\alpha\} - \alpha \tag{1}$$

The geometry of the street-of-coverage for a single-orbit plane containing S symmetrically distributed satellites (three or more) is shown in Fig. 2. The relation between half-street width C_j , the coverage angle θ , and the number of satellites in the orbit S is given by S

$$C_j = \cos^{-1} \left[\frac{\cos \theta}{\cos(j\pi/S)} \right] \tag{2}$$

There are two types of interfaces between orbit planes. The first, called a corotating interface, is when the satellites in the two planes are moving in the same relative direction along the coverage interface. The second, called a counter-rotating interface, is when the satellites in the two planes move in opposite directions along the coverage interface. A constellation with a maximal number of corotating interfaces is called a cooriented constellation. It is known that cooriented constellations are better than counteroriented constellations for global coverage. ¹⁴ Thus, the cooriented polar constellations

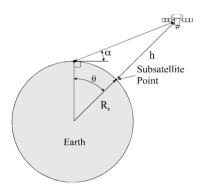


Fig. 1 Coverage geometry.

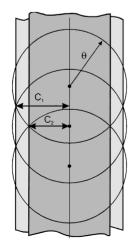


Fig. 2 Streets-of-coverage.

will be considered as a basis. The constraint relation for multiple coverage can be written as^{12}

$$(P-1)(\theta + C_i) \ge k\pi - (C_1 + C_i) \tag{3}$$

The maximum spacing between corotating optimally phased orbit planes is $\Delta\lambda=\theta+C_j$, and the maximum spacing between counterrotating orbit planes is $\Delta\lambda^*=\pi-(C_1+C_j)$ (Ref. 12). It should be noted that in some cases the last relation for counter-rotating interfaces is conservative.¹⁴

Meshing Geometry of Near-Polar Satellite Constellations

Continuous Single-Global Coverage

Let us consider a single-coverage constellation (j=1 and k=1). The orbit inclination and the ascending nodes for the first and second orbit planes (corotating) can be arranged so that the angle between them is $\theta+C_1$. The interplane satellite phasing can be selected to provide continuous coverage of the region that lies between these planes (Fig. 3). The maximum angular separation between the ascending nodes of these orbit planes is

$$\Delta \lambda_1 = 2 \sin^{-1} \left\{ \frac{\sin[(\theta + C_1)/2]}{\sin i} \right\} \ge (\theta + C_1) \tag{4}$$

The ascending node of each successive corotating plane can be placed to ensure a continuous coverage interface with the previous orbit plane. In general, it is clear that inclinations less than $i_{\min} \cong \pi/2 - C_1$ cannot provide continuous coverage near the poles. (This approximation is exact for P + S odd. For P + S even, this relationship is conservative.¹⁴)

The critical point that determines the maximum separation between the ascending nodes of the first and Pth orbital planes (counter-rotating) depends on the orbit inclination. Examples of the meshing geometry for polar inclination, $\pi/2 > i > i_{\min}$, and i_{\min} are shown in Figs. 4, 5, and 6, respectively. In Figs. 4-6, the outer circle represents the equator, and the pole is located at the center of the circle. The meridians are straight lines through the pole. The orbit traces (relative to the fixed Earth) are boldface lines (arrows are associated with the direction of motion). The streets-of-coverage are shaded areas. To provide continuous coverage along the counter-rotating interface between the first and Pth planes, it is necessary to have

$$\Delta \lambda_1^* = \cos^{-1} \left[\frac{\cos(\pi - 2C_1) - \cos^2 i}{\sin^2 i} \right]$$
 (5)

In the case of polar orbits, this angle will have the minimum value of $\Delta \lambda_1^* = \pi - 2C_1$. For the minimum inclination i_{\min} , angular separation $\Delta \lambda_1^*$ will attain the maximum value equal to π .

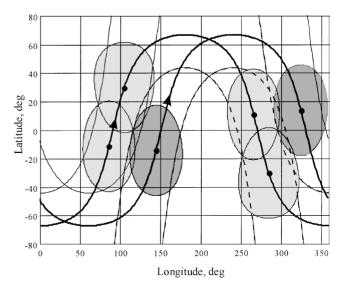


Fig. 3 Phasing geometry for continuous single coverage.

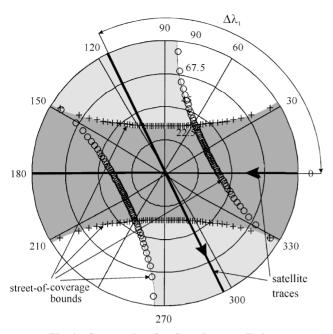


Fig. 4 Coverage interface for polar constellations.

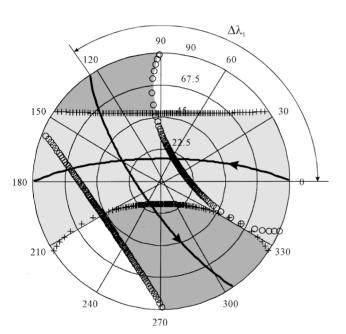


Fig. 5 Coverage interface for $\pi/2 > i > i_{\min}$.

As a result, the constraint relation for inclined constellations can be given as

$$(P-1)\Delta\lambda_1 \ge \Delta\lambda_1^* \tag{6}$$

The interplane satellite phasing is given by

$$\Delta \varphi^* = \frac{\Delta \varphi}{2} - 2 \cos^{-1} \left\{ \frac{\cos(\Delta \lambda_1/2)}{\cos[(\theta + C_1)/2]} \right\}$$
 (7)

Continuous Double-Global Coverage

A similar generalization of polar systems can be obtained for continuous double-global coverage. There are two types of meshing geometry.

For the first type of constellations (j=1 and k=2), the ascending nodes of all orbit planes can be distributed over 2π rad, with an approximate value for $\Delta\lambda$ being $2\pi/P$. Each orbit plane must provide a single street-of-coverage. In this case, we have a corotating

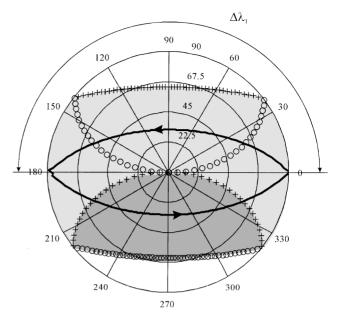


Fig. 6 Coverage interface for $i = i_{min}$.

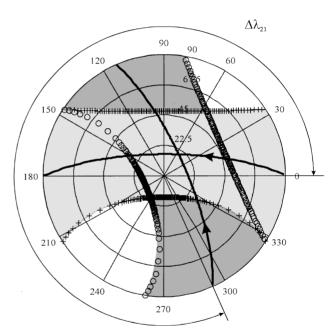


Fig. 7 Interface for first type of double coverage.

interface between the first and *P*th planes (Fig. 7). The necessary angular separation between ascending nodes of these orbits is

$$\Delta \lambda_{21}^* = 2\pi - \cos^{-1} \left[\frac{\cos(2\pi - 2C_1) - \cos^2 i}{\sin^2 i} \right]$$
 (8)

and the constraint relation can be written as

$$(P-1)\Delta\lambda_1 \ge \Delta\lambda_{21}^* \tag{9}$$

The interplane satellite phasing is given by Eq. (7).

For the second type of constellations (j=2 and k=1), the ascending nodes of all orbit planes with a double street-of-coverage can be distributed over π rad, with an approximate value for $\Delta\lambda$ being π/P . As is known, the angle between corotating, phased orbits providing continuous double coverage should be no less than $\theta+C_2$ (Refs. 12, 14, and 16) (Fig. 8). In the case of a counter-rotating

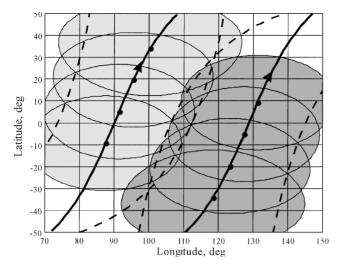


Fig. 8 Phasing geometry for continuous double coverage.

interface, the necessary angle between the two orbit planes is $C_1 + C_2$. Finally, for near-polar orbits, the required separation angles can be given as

$$\Delta \lambda_2 = 2 \sin^{-1} \left\{ \frac{\sin[(\theta + C_2)/2]}{\sin i} \right\} \ge (\theta + C_2)$$
 (10)

$$\Delta \lambda_{22}^* = \cos^{-1} \left[\frac{\cos(\pi - C_1 - C_2) - \cos^2 i}{\sin^2 i} \right]$$
 (11)

and the constraint relation can be written as

$$(P-1)\Delta\lambda_2 \ge \Delta\lambda_{22}^* \tag{12}$$

The expression for the interplane phasing angle may be written as

$$\Delta \varphi^* = \Delta \varphi - 2 \cos^{-1} \left\{ \frac{\cos(\Delta \lambda_2/2)}{\cos[(\theta + C_2)/2]} \right\}$$
 (13)

The minimum inclination for these constellations is $i_{\min} \cong \pi/2 - C_2$.

Near-Polar Constellations

The design of practical satellite constellations relies on the analysis of a number of mutually linked factors, such as minimization of total number of satellites, type of launch service, orbital perturbations, type of satellites, etc. Constellation catalogs are most often used as an initial basis for selecting a constellation. Such catalogs can be presented for various constellation types (polar¹⁴ or inclined, ^{13,15,22,23} phased ^{14,22,23} or unphased, ^{13,14} symmetrical ^{15,22,23} or nonsymmetrical, ^{13,14} with specified orbit inclination, ²¹ and with all numbers of orbit planes ^{15,22,23}).

Next, a brief survey of the near-polar constellation set and a comparison of it to other constellation families and catalogs will be presented.

Minimal Constellations

The determination of a set of near-polar constellations with the minimum number of satellites can be determined by known iterative computational procedures ¹⁴ that calculate minimum θ values that satisfy the aforementioned constraint relations. These procedures are incorporated into a minimal constellation selection process as follows: Potential values of N are examined in increasing order. For a selected value of N, all factorizations of $N = P \times S$ are considered. When double coverage is to be considered, both constellation types are examined. The minimum value for θ must satisfy the appropriate constraint relation. If the value of θ so determined is larger than a previous value of θ computed either for a smaller value of N or for a different factorization of N, then the constellation cannot be minimal and is discarded. If, on the other hand, the value of θ is smaller than that determined for a previous constellation consisting

of N satellites, then the previous constellation cannot be minimal and is discarded. The constellations that remain after this discarding process are precisely the minimal constellation as a function of θ (or orbit altitude h) (Ref. 14).

Single-Global Coverage Constellations

There are two effects related to an inclination change from a polar to a near-polar orbit. There is an increase in the angular separation between the ascending nodes for corotating orbital planes, which determined the left-hand side of Eq. (6). Likewise, there is an increase in the angular separation between the ascending nodes of the first and Pth orbital planes. In all cases, except the two-plane constellations, $\Delta \lambda_1^*$ increases faster than $(P-1)\Delta \lambda_1$. Therefore, the all-polar-phased constellations are more efficient than the similar near-polar constellations for single-global coverage.

The minimum θ (and, respectively, h) for the similar two-plane satellite constellations remains constant for any of the possible values of inclination, including polar orbits. This phenomenon occurs inasmuch as Eq. (6) is independent of orbit inclination because from Eq. (3) for the optimal two-plane polar constellations we have

$$3C_1 + \theta = \pi \tag{14}$$

In fact, it has been confirmed by substitution that this solution satisfies Eq. (6) for arbitrary $\pi/2 \ge i \ge i_{\min}$.

The minimal constellations for $i = i_{\min}$ and $i = \pi/2$ are shown in Table 1.

The dependence of the minimum orbit altitude on inclination is a monotonic function (see examples in Fig. 9). It is shown that the altitude ranges for low Earth orbits are small and comparable to altitude variations due to small eccentricity and other orbital perturbations. In the case of $i = 86.4 \, \text{deg}$, $\alpha = 8.2 \, \text{deg}$, and $N = 6 \times 11$, this method yielded a satellite constellation very close to the IRIDIUM constellation.

Double-Global Coverage Constellations

As already mentioned, there are two types of the near-polar constellations. Of special interest is the first type, which produced better results than similar polar constellations. With inclination decreasing, the angular separation between the ascending nodes of the first and Pth orbital planes decreased. Therefore, from Eq. (9) it follows that the first type, i.e., j = 1 and k = 2, of near-polar constellations are always better than the similar polar constellations. The opposite situation occurs with the second type, i.e., j = 2 and k = 1, of constellations. For similar two-plane constellations, the same minimum altitude that applies to a phased-polar constellation will also be applicable to the near-polar constellation for any allowable inclination. For P > 2 the polar-phased constellations are more efficient than the similar near-polar constellations. For a specified total number of satellites, the best constellation depending on the inclination may be any one of the types. Examples of minimal altitudes for some constellations with 48 and 64 satellites are shown in Fig. 10.

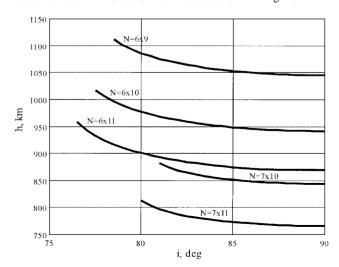


Fig. 9 Orbit altitudes of near-polar constellations for single coverage.

Table 1	Set of near-polar satellit	e constellations f	for single-globa	lcoverage
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N	P	S	Coverage angle range, deg	Inclination range, deg	Range for $\Delta \lambda$, deg	Range for $\Delta \varphi^*$, deg	Altitude range, km $(\alpha = 10 \text{ deg})$
6	2	3	66.72	52.24-90	180-104.5	0-60	20,959
8	2	4	57.63	49.21-90	180-98.4	45-45	10,127
10	2	5	53.22	47.74-90	180-95.5	0-36	7,563
15	3	5	45.51-42.28	60.02-90	90-66.1	54.9-36	4,715-3,889
18	3	6	42.46-38.68	58.42-90	90-64.3	34.8-30	3,930-3,136
20	4	5	39.49-38.03	72.54-90	60-51.2	16.4-36	3,292-3,015
24	4	6	35.66-33.58	69.74-90	60-49.4	7.4-30	2,610-2,293
28	4	7	33.31-30.78	68.07-90	60-48.3	1.4-25.7	2,254-1,918
32	4	8	31.75-28.91	66.98-90	60-47.6	42.1-22.5	2,042-1,695
35	5	7	29.19-27.96	75.68-90	45-39.3	14-25.7	1,727-1,589
40	5	8	27.30-25.74	74.12-90	45-38.6	9.6-22.5	1,518-1,361
45	5	9	25.99-24.18	73.06-90	45-38.1	6.2-20	1,385-1,215
48	6	8	24.75-24.00	79.40-90	36-32.6	15.7-22.5	1,267-1,199
54	6	9	23.19-22.21	78.01-90	36-32.1	12.3-20	1,128-1,046
60	6	10	22.06-20.90	77.03-90	36-31.7	9.7-18	1,034-942
66	6	11	21.22-19.91	76.30-90	36-31.4	7.6-16.4	968-869
70	7	10	20.22-19.57	80.64-90	30-27.4	13-18	891-845
77	7	11	19.24-18.46	79.74-90	30-27.1	10.9-16.4	821-767
84	7	12	18.49-17.60	79.06-90	30-26.9	9.2-15	769-710
88	8	11	17.97-17.53	82.48-90	25.7-23.9	12.9-16.4	734-706
91	7	13	17.91-16.92	78.53-90	30-26.7	7.7-13.8	730-667
96	8	12	17.11-16.57	81.67-90	25.7-23.7	11.2-15	679-645

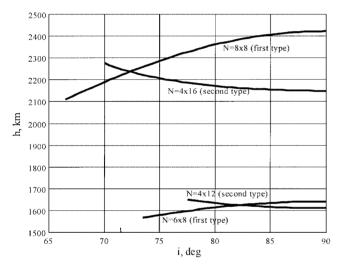


Fig. 10 Orbit altitudes of near-polar constellations for double coverage.

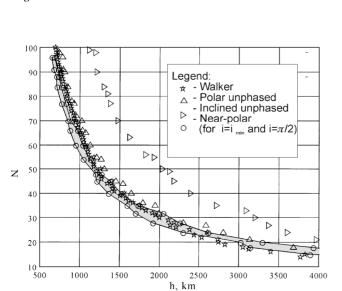


Fig. 11 Comparison of single-coverage constellations, $\alpha = 10$ deg.

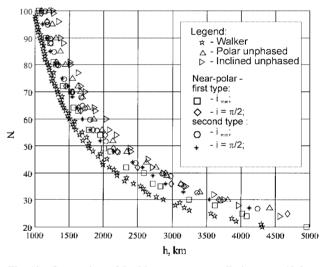


Fig. 12 Comparison of double-coverage constellations, α = 10 deg.

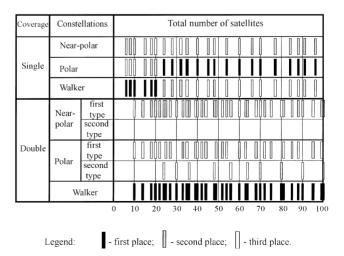
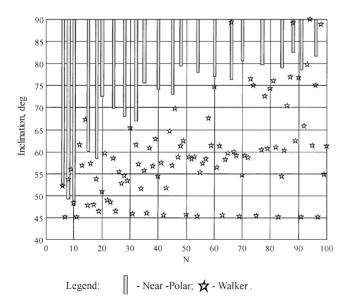


Fig. 13 Schematic diagram for distributions of best constellations.

Table 2 Satellite constellations for double-global coverage

	Table 2 Sateline constellations for double-global coverage									
	D	G	θ ,	i,	Δλ,	$\Delta \varphi^*$,	<i>h</i> , km			
N	P	S	deg	deg	deg	deg	$(\alpha = 10 \text{ deg})$			
10	2	5	73.02	70.93	99.23	29.98	45,301			
12	2	6	63.89	61.67	109.91	51.84	16,256			
14	2	7	58.61	56.65	118.84	16.98	10,847			
16	2	8	55.30	53.63	125.97	36.37	8,651			
18	2	9	53.07	51.66	131.73	11.68	7,491			
20	4	5	46.36	58.54	94.85	48.80	4,956			
24	4	6	43.14	57.41	93.51	30.40	4,088			
25	5	5	42.86	64.97	76.08	71.37	4,020			
28	4	7	41.13	56.72	92.65	17.36	3,627			
30	5	6	39.27	63.37	74.96	52.07	3,245			
33	3	11	38.84	90	61.05	0	3,166			
35	5	7	37.03	62.39	74.23	38.48	2,834			
36	6	6	36.34	68.46	62.7	4.78	2,717			
39	3	13	36.32	90	60.81	0	2,716			
40	5	8	35.54	61.74	73.74	28.39	2,587			
42	6	7	33.87	67.16	62.04	50.86	2,332			
48	6	8	32.22	66.31	61.59	40.56	2,101			
49	7	7	31.48	71.18	53.36	7.3	2,004			
52	4	13	31.07	90	45.77	0	1,954			
54	6	9	31.07	65.72	61.27	32.62	1,951			
56	7	8	29.68	70.12	52.93	3.28	1,781			
60	4	15	28.94	90	45.6	0	1,697			
63	9	7	28.37	77.58	41.82	16.32	1,632			
64	8	8	27.74	73.34	46.45	8.47	1,563			
68	4	17	27.52	90	45.51	0	1,541			
70	5	14	27.26	90	36.64	0	1,514			
72	9	8	26.26	76.10	41.42	12.12	1,410			
75	5	15	26.07	90	36.57	0	1,393			
80	5	16	25.10	90	36.52	0	1,299			
81	9	9	24.77	75.08	41.14	8.96	1,267			
85	5	17	24.30	90	36.49	0	1,225			
88	11	8	24.28	80.64	34.1	16.79	1,222			
90	10	9	23.53	77.34	37.12	11.58	1,156			
95	5	19	23.04	90	36.41	0	1,115			
99	11	9	22.56	79.33	33.83	13.56	1,074			
100	10	10	22.36	76.50	36.92	9.09	1,057			



 $Fig.\ 14\quad Distributions\ of\ inclinations\ for\ single-coverage\ constellations.$

The best satellite constellations for up to 100 satellites are listed in Table 2. Most of the constellations listed are of the near-polartype with inclination i_{\min} . The four families of minimal constellations of the first and second types for $i=\pi/2$ and $i=i_{\min}$ are given in the Appendix.

Comparison with Prior Results

Comparison results in terms of total number of satellites vs orbit altitude are shown in Figs. 11 and 12 for the near-polar (shaded area), best Walker^{15,22} and unphased (polar and inclined)^{13,14} constella-

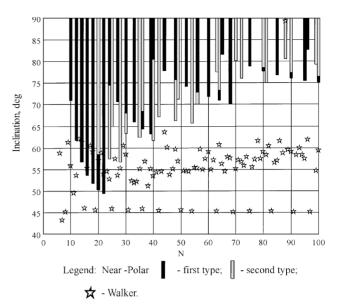


Fig. 15 Distributions of inclinations for double-coverage constellations.

tions. The distributions of the best constellations vs total number of satellites are presented in Fig. 13.

The presented data and other results^{13,22} indicate that the best Walker constellations are more efficient than the polar-phased and near-polar constellations for single coverage when the total number of satellites is less than approximately 20. For values of N greater than 20, the better systems are the polar-phased constellations. However, for N greater than approximately 35, most of the near-polar constellations are better than the best Walker constellations. For double coverage, the best Walker constellations head the list for all considered values of N. Notice that the best near-polar constellations for $i = i_{\min}$ belong only to the first type of constellations.

The best Walker and near-polar constellations formed, as a rule, two nonoverlapping sets of orbit inclinations. Joint distributions of them for single and double coverage are presented in Figs. 14 and 15, respectively. It is shown that, in a sense, these constellation types complement each other well.

Conclusions

We have presented a new set of near-polar satellite constellations in circular orbits for continuous single- and double-global coverage based on the generalization of known polar-phased constellations. In some design situations nonpolar orbits can be favored.

There are at least three interesting results. First, and most important, there is a possibility of being able to select a wide range of orbit inclination with slight changes of orbit altitudes. Second, this method produced better constellations than the original polar constellations for double coverage. Last, for any of the allowable inclinations, the two-plane near-polar constellations can be used for the same orbit altitude as the similar polar constellations. Although the new constellations are not as efficient as the best constellations known to the author, i.e., the polar-phased constellations with N > 20 for single coverage and the Walker-type in all other cases, they are applicable to orbit inclinations different from those used by the more efficient constellations.

Similar to other research, a more complex comparison, e.g., launch vehicle capability, crosslinking, sparing strategy, etc., between the near-polar constellations with all numbers of orbit planes and the Walker constellations may be very useful. Some constellations for single-global coverage can be improved using a more efficient relationship for phased counter-rotating interfaces, as in earlier research. We hope that a similar generalization of polar-phased constellations can be used to obtain the near-polar constellations for triple and higher orders of coverage.

Appendix: Boundary Families for Double Coverage

Table A1 summarizes some parameters of the boundary families of the minimal constellations for double continuous coverage.

Table A1 Families of minimal constellations for double-global coverage

	First type							Second type				
			μ,	deg	$h, \text{km} (\alpha = 10 \text{ deg})$				μ , deg		$h, \text{km} (\alpha = 10 \text{ deg})$	
N	P	S	i_{\min}	$\pi/2$	i_{\min}	$\pi/2$	P	S	i_{\min}	$\pi/2$	i_{\min}	$\pi/2$
10	a						2	5	73.02	73.02	45,301	45,301
12							2	6	63.89	63.89	16,256	16,256
14							2	7	58.61	58.61	10,847	10,847
15	5	3		61.76		13,670						
16							2	8	55.3	55.3	8,651	8,651
18 20 ^b	4		46.26		4.056		2	9	53.07	53.07	7,491	7,491
20 ^b	5	5 4	46.36	50.64	4,956	6,425	2	10	51.5	51.5	6,786	6,786
22		4		30.04		0,423	2	11	50.35	50.35	6,319	6,319
24	4	6	43.14		4,088		3	8	47.05	46.85	5,170	5,107
24	6	4		47.58		5,332						
25	5	5	42.86	45.38	4,020	4,673						
27							3	9	43.71	43.27	4,233	4,124
28	4	7	41.13		3,627							
30	5	6	39.27		3,245		3	10	41.42	40.72	3,694	3,543
30	6	5		41.36		3,676						
33			27.02		2.024		3	11	39.81	38.84	3,355	3,166
35 35	5 7	7 5	37.03	38.89	2,834	3,172						
36	6	6	36.34	37.91	2,717	2,990	3	12	38.64	37.42	3,128	2,905
39			30.34	37.91	2,/1/	2,990	3	13	37.77	36.32	2,968	2,716
42	6	7	33.87		2,332		3	14		35.44		2,574
42	7	6		34.94		2,492						
44							4	11	34.69	34.46	2,457	2,423
48	6	8	32.22		2,101		4	12	32.93	32.56	2,200	2,149
48	8	6		32.97		2,204						
49	7	7	31.48	32.51	2,004	2,141						
52			21.07		1.051		4	13	31.6	31.07	2,021	1,954
54	6	9	31.07		1,951	2.020						
54 56	9 7	6 8	29.68	31.67	1,782	2,029	4	14	30.57	29.89	1,892	1,808
56	8	7	29.06	30.24	1,/62	1,849	-4		30.37	29.09	1,092	1,000
60							4	15	29.78	28.94	1,795	1,697
63	9	7	28.37	28.65	1,632	1,663						
64	8	8	27.74	28.45	1,563	1,640	4	16	29.15	28.16	1,722	1,610
65						<u>-</u>	5	13	28.83		1,685	
68							4	17	28.65	27.52	1,665	1,541
70	10	7	27.4	27.54	1,527	1,542	5	14	27.44	27.26	1,533	1,514
72	9	8	26.26	26.66	1,410	1,450						
75							5	15	26.34	26.07	1,420	1,393
80	10	8	25.13	25.36	1,300	1,322	5	16	25.47	25.1	1,334	1,299
81 85	9	9	24.77	25.28	1,267	1,315	5	17	24.76	24.2	1 267	1 225
88 88	11	8	24.28	24.4	1,222	1,233			24.70 ——	24.3	1,267	1,225
90	10	9	23.53	23.83	1,156	1,182	5	18	24.17	23.61	1,214	1,164
90							5	18		23.61		1,164
95							5	19	23.69	23.04	1,171	1,115
96							6	16	23.60		1,163	
99	11	9	22.56	22.75	1,074	1,089						
100	10	10	22.36	22.74	1,057	1,088	5	20	23.29	22.56	1,136	1,074

^aDash denotes that a constellation is infeasible or is not a minimal constellation.

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^bFor the same value of N, there are minimal constellations with distinct numbers of orbit planes. The first line indicates a constellation with $i = i_{\min}$, and the second line indicates a constellation with $i = \pi/2$.

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